



$$\begin{vmatrix} 1 & -1 & 1 \\ \alpha & 2 & -1 \\ 8 & \alpha - 6 & 3 \end{vmatrix} = 0$$

$$1(6 + \alpha - 6) + 1(3\alpha + 8) + 1(\alpha^2 - 6\alpha - 16) = 0$$

$$\alpha^2 - 2\alpha - 8 = 0$$

$$(\alpha - 4)^2 = 9 \Rightarrow \alpha = 4, -2$$

$$\text{Sum} = 2$$

10. Find rank of the word "MOTHER" in the dictionary.
 (A) 310 (B) 309 (C) 308 (D) 311

Ans. (B)

Sol. M O T H E R
 2 2 3 1 0 0

$$5!_0 4!_0 3!_0 2!_0 1!_0 0!_0$$

$$5!_0 \times 2 + 4!_0 \times 2 + 3!_0 \times 3 + 1!_0 \times 2$$

$$\Rightarrow 240 + 48 + 10 + 2$$

$$\Rightarrow 260 + 48 = 308 + 1 = 309$$

11. Find the value of $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$:
 (A) 5 (B) 4 (C) 3 (D) 2

Ans. (B)

Sol. $(\tan 9^\circ + \cot 9^\circ) - (\tan 27^\circ + \cot 27^\circ)$

$$= \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ} \quad \{ \tan \theta + \cot \theta = 2 \operatorname{cosec} 2\theta \}$$

$$= \frac{8}{\sqrt{5}-1} - \frac{8}{\sqrt{5}+1} = \frac{8(\sqrt{5}+1-\sqrt{5}+1)}{4}$$

$$= 4$$

12. Find the square of distance of point (12, 12, 18) from a plane which passes through line of intersection of planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$ and also passes through point (0, 2, -2).

Ans. 620

Sol. $x + y + z = 6$

$$2x + 3y + 4z = -5$$

$$(x + y + z - 6) + \lambda(2x + 3y + 4z + 5) = 0$$

$$\Rightarrow (2-2)-6 + \lambda(0 + 6 - 8 + 5) = 0$$

$$-6 + 3\lambda = 0 \quad (\lambda = 2)$$

$$(x + y + z - 6) + 4x + 6y + 8z + 10 = 0$$

$$\Rightarrow 5x + 7y + 9z + 4 = 0$$

$$\Rightarrow \text{Distance} = \left| \frac{5 \times 12 + 7 \times 2 + 9 \times 18 + 4}{\sqrt{25 + 49 + 81}} \right| = \frac{310}{\sqrt{155}}$$

$$\text{Answer} = \left(\frac{310}{\sqrt{155}} \right)^2 = 620$$

13. Given that
 $20^{19} + 2 \cdot 21 \cdot (20)^{18} + 3(21)^2(20)^{17} + 4(21)^3(20)^{16} + \dots + 20(21)^{19} = S$
 Find the value of S:





Ans. $(20)^{21}$

Sol. $S = 20^{19} + 2 \cdot 21(20)^{18} + 3(21)^2 (20)^{17} + \dots + 20(21)^{19}$

$$\frac{S}{(21)^{19}} = \left(\frac{20}{21}\right)^{19} + 2\left(\frac{20}{21}\right)^{18} \times 3\left(\frac{20}{21}\right)^{17} + \dots + 20$$

Let $\frac{S}{(21)^{19}} = k$

So, $k = \left(\frac{20}{21}\right)^{19} + 2 \cdot \left(\frac{20}{21}\right)^{18} + 3\left(\frac{20}{21}\right)^{17} + \dots + 20$

$$\frac{20k}{21} = 20\left(\frac{20}{21}\right)^{19} + 19\left(\frac{20}{21}\right)^{18} + \dots + 2\left(\frac{20}{21}\right)^{17} + \left(\frac{20}{21}\right)^{16}$$

$$\frac{k}{21} = 20 - \left(\frac{20}{21}\right)^{19} - \left(\frac{20}{21}\right)^{18} - \dots - \left(\frac{20}{21}\right)^{16}$$

$$\Rightarrow \frac{k}{21} = 20 - \left[\frac{20}{21} \left(\frac{1 - \left(\frac{20}{21}\right)^{20}}{1 - \frac{20}{21}} \right) \right] = \frac{k}{21} = \frac{20(20)^{20}}{(21)^{20}}$$

$$\Rightarrow \frac{S}{(21)^{19} \times 21} = \frac{20^{21}}{21^{20}} \Rightarrow S = (20)^{21}$$

14. Check the statements

Statement: 1 $(p \Rightarrow q) \vee (\sim p \wedge q)$: tautology

Statement: 2 $(q \Rightarrow p) \Rightarrow (\sim p \wedge q)$: contradiction

(A) Both are true

(B) Neither 1 nor 2 are true

(C) Only First One is true

(D) Both are false

Ans. (B)

Sol. $(\sim p \vee q) \vee (\sim p \wedge q)$

$$\equiv \sim p \vee q$$

$$\sim(\sim q \vee p) \vee (\sim p \wedge q)$$

$$\equiv (\sim p \wedge q) \vee (\sim p \wedge q)$$

$$\equiv \sim p \wedge q.$$

Hence neither is true.

15. Check the statements

S_1 : $(2023)^{2022} - (1999)^{2022}$ is divisible by 8

S_2 : $13(13)^n - 11n - 13$ is divisible by 144 for infinite values of $n \in \mathbb{N}$.

(A) S_1 & S_2 both are correct

(B) S_1 & S_2 both are incorrect

(C) S_1 is correct S_2 is incorrect

(D) S_1 is incorrect & S_2 is correct

Ans. (A)

Sol. S_1 : Note that $a^n - b^n$ is divisible by $a - b$

Hence $(2023)^{2022} - (1999)^{2022}$ is divisible by 24 hence by 8

S_2 : $13(12 + 1)^n - 11n - 13$

$$= 13 \left[{}^n C_0 12^n + \dots + {}^n C_{n-2} 12^2 + {}^n C_1 12 + 1 \right] - 11n - 13$$

$$= 144k + (156 - 11)n$$

$$= 144k + 145n$$





Hence if n is a multiple 144 it is divisible by 144
So infinite such n exists.

16. The system of equations.
 $P_1: x + y + z = 6$
 $P_2: x + 2y + \alpha z = 5$
 $P_3: x + 2y + 6z = \beta$ has
 (A) Infinitely many solutions $\alpha = 6, \beta = 3$
 (B) Infinitely many solutions $\alpha = 6, \beta = 5$
 (C) Unique's solutions $\alpha = 6, \beta = 5$
 (D) No solutions $\alpha = 6, \beta = 5$

Ans. (B)

Sol. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & \alpha \\ 1 & 2 & 6 \end{vmatrix}, \Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 5 & 2 & \alpha \\ \beta & 2 & 6 \end{vmatrix}$

$$\Delta = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & \alpha - 1 \\ 1 & 1 & 6 - 1 \end{vmatrix}, \Delta_1 = \begin{vmatrix} 0 & 0 & 1 \\ -7 & 2 - \alpha & \alpha \\ \beta - 12 & -4 & 6 \end{vmatrix}$$

$$\Delta = 5 - \alpha + 1 \quad \Delta_1 = 1[28 - (2 - \alpha)(\beta - 12)]$$

$$\Delta = 6 - \alpha \quad \Delta_1 = 28 + (\alpha - 2)(\beta - 12)$$

For infinite many solution $\Delta = 0, \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = 0$$

$$\alpha = 6$$

$$\text{When } \Delta_1 = 0, \Rightarrow 28 + (6 - 2)(\beta - 12) = 0$$

$$\Rightarrow 4(\beta - 12) = -28$$

$$\Rightarrow \beta = 12 - 7 = 5$$

$$\text{Now } \Delta_2 = \begin{vmatrix} 1 & 6 & 2 \\ 1 & 5 & 6 \\ 1 & \beta & 6 \end{vmatrix} = -4(\beta - 5)$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 5 \\ 1 & 2 & \beta \end{vmatrix} = \begin{vmatrix} 1 & 1 & 6 \\ 0 & 1 & -1 \\ 1 & 1 & \beta - 6 \end{vmatrix} = \beta - 5$$

Hence, at $\alpha = 6$ & $\beta = 5$

$$\Delta = 0, \text{ \& } \Delta_1 = \Delta_2 = \Delta_3 = 0$$

Thus at $\alpha = 6$ & $\beta = 5$ system of equation has infinite solutions.

17. R
 (A) 5 (B) 5 (C) 5 (D) 5

Ans. ()

Sol.

18. R
 (A) 5 (B) 5 (C) 5 (D) 5

Ans. ()

Sol.

19. R
 (A) 5 (B) 5 (C) 5 (D) 5

Ans. ()





Sol.

20. R
(A) 5 (B) 5 (C) 5 (D) 5

Ans.
Sol.
()

21. R
(A) 5 (B) 5 (C) 5 (D) 5

Ans.
Sol.
()

22. R
(A) 5 (B) 5 (C) 5 (D) 5

Ans.
Sol.
()

23. R
(A) 5 (B) 5 (C) 5 (D) 5

Ans.
Sol.
()

24. R
(A) 5 (B) 5 (C) 5 (D) 5

Ans.
Sol.
()

25. R
(A) 5 (B) 5 (C) 5 (D) 5

Ans.
Sol.
()

26. R
(A) 5 (B) 5 (C) 5 (D) 5

Ans.
Sol.
()

27. R
(A) 5 (B) 5 (C) 5 (D) 5

Ans.
Sol.
()

28. R
(A) 5 (B) 5 (C) 5 (D) 5

Ans.
Sol.
()

29. R
(A) 5 (B) 5 (C) 5 (D) 5

Ans.
Sol.
()





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30. R
(A) 5 (B) 5 (C) 5 (D) 5
- Ans.** ()
- Sol.**



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