

JEE-Main-27-01-2024 (Memory Based)

[EVENING SHIFT]

Mathematics

Question: If 20th term from the end of progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots, 129\frac{1}{4}$ is _____.

Options:

- (a) -120
- (b) -115
- (c) -125
- (d) -110

Answer: (b)

Solution:

$$\begin{aligned} T_{20} &= a + (n-1)d \\ &= -129\frac{1}{4} + 19 \times \frac{3}{4} \\ &= -115 \end{aligned}$$

Question: $P = (1-x)^{2008} (1+x+x^2)^{2007}$; find the coefficient of x^{2012} ?

Answer: 0.00

Solution:

$$\begin{aligned} &(1-x)^{2008} (1+x+x^2)^{2007} \\ &= (1-x)(1-x^3)^{2007} \\ &= (1-x) \sum_{r=0}^{2007} {}^{2007}C_r (-x^3)^{2007-r} \end{aligned}$$

Coefficient of $x^{2012} = 0$

Question: The integral $\int \frac{(x^8 - x^2)}{(x^{12} + 3x^6 + 1) \tan^{-1}\left(x^3 + \frac{1}{x^3}\right)} dx$ is equal to:

Options:

- (a) $\frac{1}{3} \ln \left| \left(\tan^{-1} \left(x^3 + \frac{1}{x^3} \right) \right) \right| + C$
- (b) $\ln \left| \left(\tan^{-1} \left(x^3 + \frac{1}{x^3} \right) \right) \right| + C$

$$(c) \frac{1}{6} \ln \left| \tan^{-1} \left(x^3 + \frac{1}{x^3} \right) \right| + C$$

$$(d) \frac{1}{9} \ln \left| \tan^{-1} \left(x^3 + \frac{1}{x^3} \right) \right| + C$$

Answer: (a)

Solution:

$$\begin{aligned} & \int \frac{(x^8 - x^2)}{(x^{12} + 3x^6 + 1) \tan^{-1} \left(x^3 + \frac{1}{x^3} \right)} dx \\ &= \int \frac{x^6 \left(x^2 - \frac{1}{x^4} \right)}{x^6 \left(x^6 + 3 + \frac{1}{x^6} \right) \tan^{-1} \left(x^3 + \frac{1}{x^3} \right)} dx \\ &= \int \frac{\left(x^2 - \frac{1}{x^4} \right) dx}{\left(\left(x^3 + \frac{1}{x^3} \right)^2 + 1 \right) \tan^{-1} \left(x^3 + \frac{1}{x^3} \right)} \end{aligned}$$

$$\text{Put } \tan^{-1} \left(x^3 + \frac{1}{x^3} \right) = t$$

$$\Rightarrow \frac{1}{1 + \left(x^3 + \frac{1}{x^3} \right)^2} \times 3 \left(x^2 - \frac{1}{x^4} \right) dx = dt$$

$$\Rightarrow \frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \ln \left| \tan^{-1} \left(x^3 + \frac{1}{x^3} \right) \right| + C$$

Question: $S_n = 2023\alpha^n + 2024\beta^n$, $x^2 - x - 1 = 0$

Options:

(a) $2S_{12} = S_{11} + S_{10}$

(b) $S_{12} = S_{10} + S_{11}$

(c) $S_{12} = S_{10} + S_{11}$

(d) $2S_{12} = S_{10} + S_{11}$

Answer: (a)

Solution:

$$ax^2 + bx + c = 0$$

$$aS_n + bS_{n-1} + cS_{n-2} = 0$$

$$S_{12} - S_{11} - 1 \times S_{10} = 0$$

Question: Consider an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$); eccentricity e_1 & Hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$, with eccentricity e_2 , $e_1 e_2 = 1$. Ellipse passes through the foci of hyperbola find the length of the ellipse along $y = 2$.

Answer: $\frac{10\sqrt{5}}{3}$

Solution:

Hyperbola

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$e_1 = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$$

For ellipse

$$2a = 2 \times 4e_1$$

$$a = 4e_1$$

$$a = 5$$

Also

$$e_1 e_2 = 1$$

$$\frac{5}{4} e_2 = 1$$

$$e_2 = \frac{4}{5}$$

By ellipse formula

$$b^2 = a^2 (1 - e_2^2)$$

$$b = 3$$

$$\text{Ellipse : } \frac{x^2}{25} + \frac{y^2}{9} = 1$$

Put $y = 2$ in ellipse equation

$$x_1^2 = \frac{25 \times 5}{9}$$

$$x_1 = \frac{5\sqrt{5}}{3}$$

Length of the chord

$$\Rightarrow l = 2x_1 = \frac{10\sqrt{5}}{3}$$

Question: If $2 \tan^2 \theta - 5 \sec \theta = 1$ has exactly 7 solutions in $\left[0, \frac{n\pi}{2}\right]$ least value of $n \in N$,

then $\sum_{k=1}^n \frac{k}{2^n}$ is equal to _____.

Options:

(a) $\frac{9}{2^9}$

(b) $\frac{91}{2^{13}}$

(c) $\frac{7}{2^7}$

(d) $\frac{11}{2^{12}}$

Answer: (b)

Solution:

$$2 \tan^2 \theta - 5 \sec \theta = 1$$

$$2(\sec^2 \theta - 1) - 5 \sec \theta = 1$$

$$2 \sec^2 \theta - 5 \sec \theta - 3 = 0$$

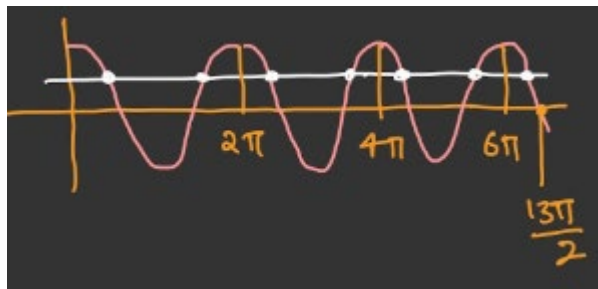
$$\Rightarrow (2 \sec \theta + 1)(\sec \theta - 3) = 0$$

$$\Rightarrow \sec \theta = \frac{-1}{2}, \sec \theta = 3$$

$$\cos \theta = -2, \cos \theta = \frac{1}{3}$$

7 solutions in $\left[0, \frac{n\pi}{2}\right]$

$$n = 13$$



$$\sum_{k=1}^{13} \frac{k}{2^{13}} = \frac{1}{2^{13}}(1+2+\dots+13)$$

$$= \frac{1}{2^{13}} \times \left(\frac{13 \times 14}{2}\right) = \frac{91}{2^{13}}$$

Question: $\lim_{x \rightarrow 0} \frac{3 - a \sin x - b \cos x - \log(1+x)}{3 \tan^2 x}$ is non zero finite. Find $2b - a = ?$

Answer: 7.00

Solution:

$$\lim_{x \rightarrow 0} \left(\frac{3 - a \sin x - b \cos x - \ln(1+x)}{3 \tan^2 x} \right)$$

$$\lim_{x \rightarrow 0} \left(\frac{3 - a \left(x - \frac{x^3}{6} \right) - b \left(1 - \frac{x^2}{2} \right) - \left(x - \frac{x^2}{2} \right)}{3x^2} \right)$$

$$\text{Coeff. of } x^0 : 3 - b = 0 \Rightarrow b = 3$$

$$\text{Coeff. of } x : -a - 1 = 0 \Rightarrow a = -1$$

$$\Rightarrow 2b - a = 7$$

Question: $\int_0^{\pi} \frac{dx}{1 - 2a \cos x + a^2}$ is equal to:

Options:

(a) $\frac{(1+a^2)\pi}{(1-a^2)^2}$

(b) $\frac{\pi}{(1-a^2)}$

(c) $\frac{(1-a^2)\pi}{(1+a^2)}$

(d) $\frac{(1-a^2)\pi}{(1+a^2)^2}$

Answer: (b)

Solution:

$$I = \int_0^{\pi} \frac{dx}{1 - 2a \cos x + a^2} \quad I = \int_0^{\pi} \frac{dx}{1 + 2a \cos x + a^2}$$

$$2I = \int_0^{\pi} \frac{2(1+a^2)}{(1+a^2)^2 - 4a^2 \cos^2 x}$$

$$2I = 2 \int_0^{\frac{\pi}{2}} \frac{2(1+a^2)}{(1+a^2)^2 - 4a^2 \cos^2 x}$$

$$I = \int_0^{\frac{\pi}{2}} \frac{2(1+a^2) \sec^2 x}{(1+a^2)^2 (1 + \tan^2 x) - 4a^2} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{2}{1+a^2} \right) \sec^2 x}{\tan^2 x + \left(\frac{1-a^2}{1+a^2} \right)} dx$$

Put $\tan x = t$

$$\begin{aligned}
 I &= \frac{2}{1+a^2} \int_0^{\infty} \frac{dt}{t^2 + \left(\frac{1-a^2}{1+a^2}\right)^2} \\
 &= \left(\frac{2}{1+a^2}\right) \times \left(\frac{1+a^2}{1-a^2}\right) \times \left| \tan^{-1} t \frac{(1+a^2)}{(1-a^2)} \right|_0^{\infty} \\
 &= \frac{2}{1-a^2} \times \frac{\pi}{2} \\
 &= \frac{\pi}{1-a^2}
 \end{aligned}$$

Question: If the area $\{(x, y) : 0 \leq y \leq \min(2x, 6x - x^2)\}$ is A then $12A$ ____.

Answer: 304.00

Solution:

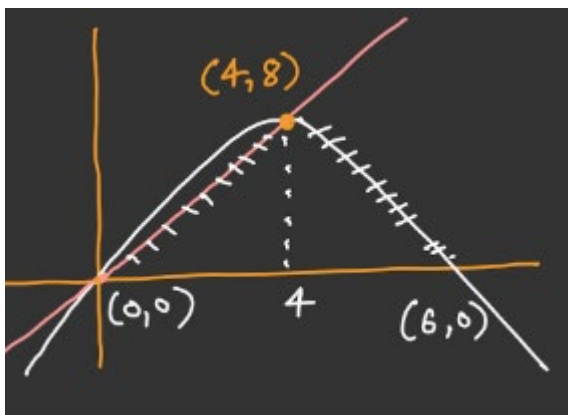
Given, $0 \leq y \leq \min(2x, 6x - x^2)$

$$6x - x^2 = 2x$$

$$x^2 = 4x$$

$$x = 0, x = 4$$

$$A = 16 + \int_4^6 (6x - x^2) dx$$



$$A = 16 + \left[3x^2 - \frac{x^3}{3} \right]_4^6$$

$$12A = 304$$

Question: If mean of 15 observation is 12 & s.d. = 3. 10 was taken wrongly in place of 12.

New mean = μ , Variance = σ^2 . Find the value = $15(\mu + \mu^2 + \sigma^2)$

Answer: 2521.00

Solution:

$$\frac{(x_1 + x_2 + \dots + x_{14}) + 10}{15} = 12 \quad \dots(1)$$

$$\frac{(x_1 + x_2 + \dots + x_{14}) + 12}{15} = \mu \quad \dots(2)$$

$$(2) - (1)$$

$$\frac{2}{15} = \mu - 12$$

$$\mu = \frac{182}{15}$$

$$\frac{x_1^2 + \dots + x_{14}^2 + 10^2}{15} - 12^2 = 9 \quad \dots(3)$$

$$\frac{x_1^2 + \dots + x_{14}^2 + 12^2}{15} - \left(\frac{182}{15}\right)^2 = \sigma^2 \quad \dots(4)$$

$$(4) - (3)$$

$$\left(\frac{12^2 - 10^2}{15}\right) - \left(\left(\frac{182}{15}\right)^2 - 12^2\right) = \sigma^2 - 9$$

$$\frac{44}{15} - \left(\frac{182}{15}\right)^2 + 144 = \sigma^2 - 9$$

$$\left(\frac{182}{15}\right)^2 + \sigma^2 = \frac{44}{15} + 144 + 9$$

$$\mu^2 + \sigma^2 + \mu = \frac{44}{15} + 153 + \frac{182}{15}$$

$$15(\mu + \mu^2 + \sigma^2) = 44 + 182 + 153 \times 15 = 2521$$

Question:
$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$$
 then α lies in

Options:

(a) $\left[-\frac{3}{2}, \frac{3}{2}\right]$

(b) $(-3, 0)$

(c)

(d)

Answer:

Solution:

$$\left(\alpha + \frac{3}{2}\right)\left[3\alpha + 1 - \frac{2\alpha}{3} - 1\right] - \left(\alpha + \frac{1}{3}\right)\left[3\alpha + 1 - 3\alpha - \frac{9}{2}\right] = 0$$

$$\left(\alpha + \frac{3}{2}\right)\frac{7\alpha}{3} + \left(\alpha + \frac{1}{3}\right)\frac{7}{2} = 0$$

$$\frac{\alpha^2}{3} + \frac{\alpha}{2} + \frac{\alpha}{2} + \frac{1}{6} = 0$$

$$2\alpha^2 + 6\alpha + 1 = 0$$

Question: Find the number of solutions: $\tan^{-1}(x) + \tan^{-1}(2x) = \frac{\pi}{4}$

Answer: 1.00

Question: A-m elements, B-n elements, subset of A is 56 more than B, P(m, n) is a point and Q(-2, -3), find distance between P and Q.

Answer: -10.00

Question: $\alpha = \frac{(4!)!}{(4!)^{3!}}$ & $\beta = \frac{(5!)!}{(5!)^{4!}}$

Options:

- (a) α is integer β is not
- (b) β is integer α is not
- (c) Both are integers
- (d) Both are not integers

Question: Let A be 2×2 real matrix and roots of equation $|A - x| = 0$ be -1 and 3. Sum of diagonal elements of A^2 ____.

Question: $g(x) = 3f\left(\frac{x}{3}\right) + f(3-x)$ and $f''(x) > 0$ for $x \in (0, 3)$