# ELECTRIC CHARGES AND FIELDS (CHAPTER-1)

CHARGE:- It is the property of the body by virtue of which it shows both electric and magnetic behaviour.

REPRESENTATION – Q or q

- Charge is a scalar quantity
- SI unit – coulomb (C)
- CGS unit – st C (electrostatic unit of charge) $1C = 3 \times 10^9$ st C
  ab C (electromagnetic unit of charge) $1C = \frac{1}{10}$ ab C

## SPECIFIC PROPERTIES OF CHARGE:-

① According to Benjamin Franklin, charges are of two types, positive and negative.

② Like charges repel and unlike charges attract. (Fundamental law of electrostatics)

③ Charge is always associated with mass.
i.e Charge cannot exist without mass whereas mass can exist without charge.

④ when a body is positively charged → lose electrons → mass decreases
when a body is negatively charged → gains electrons → mass increases

⑤ Charge is conserved:- The charge of an isolated system remains constant. That means, charge can neither be created nor be destroyed

⑥ charge is quantised:- Total charge of a body is equal to the integral multiple of fundamental charge 'e'

i.e $Q = \pm ne$ , n = an integer (1, 2, 3, ....)

★ Minimum possible charge $= \pm e = \pm 1.6 \times 10^{-19}$ C.

⑦ Charge is invariant:- Charge is independent of frame of reference. That is, charge on a body doesnot change whatever may be its speed.

⑧ charge is additive:- Total charge on an isolated system is equal to the algebraic sum of charges on individual bodies of the system.
i.e If a system contain three charges, $q_1$, $q_2$ & $q_3$ then total charge on the system $Q = q_1 + q_2 + q_3$

## Difference between charge and mass:-

| CHARGE | MASS |
|---|---|
| (1) Charge cannot exist without mass. | Mass can exist without charge. |
| (2) Force between the charges can either be attractive or repulsive. | Gravitational force between two mass is always attractive |
| (3) Charge doesnot depend on the speed of the body. | Mass of a body changes according to the formula, $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ where, c = speed of light in vacuum, m = mass of a body moving with velocity v, $m_0$ = rest mass of the body. |
| (4) Charge can be either positive, negative or zero | Mass is a positive quantity. |

## METHODS OF CHARGING

There are three methods of charging:-

1. Friction
2. Electrostatic induction
3. conduction

(1) **FRICTION**:- If we rub one body with another body, then transfer of electrons take place from one body to another body.

The transfer of $e^-$ take place from lower work function body to the higher work function body.

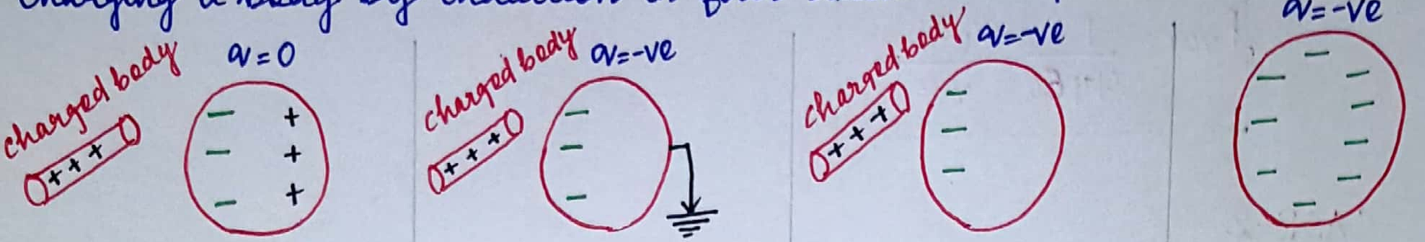
| Positive | Negative |
|---|---|
| Glass rod | Silk cloth |
| Wollen cloth | Plastic objects, rubber shoes, amber |
| Cat skin | Ebonite rod |
| Dry hair | Comb |

- clouds become charged by friction.

## (2) ELECTROSTATIC INDUCTION (without direct contact betn 2 bodies)

The phenomenon of temporary electrification of a conductor in which opposite charges appear at its closer end and similar charges appear at its farthest end in the presence of a nearby charged body is called electrostatic induction.

**Charging a body by induction in four successive steps:-**




**Step-1:-** Charged body is brought near an uncharged body.

**Step-2:-** uncharged body is connected to earth.

**Step-3:-** uncharged body is disconnected from earth.

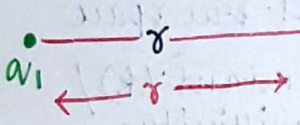
**Step-4:-** charging body is removed.

## (3) CONDUCTION:-

The process of transfer of charge by direct contact betn 2 bodies is called conduction.

## COULOMB'S LAW

The force of attraction or repulsion between any two point charges at rest is directly proportional to product of magnitude of charges and inversely proportional to square of distance between them and acts along the line joining the 2 charges.




$$F \propto q_1 q_2$$

$$F \propto \frac{1}{r^2}$$

$$\Rightarrow \boxed{F = \frac{K q_1 q_2}{r^2}}$$

where K = proportionality constant

K depends on two factors:-

(i) nature of medium between the two charges.

(ii) system of units chosen.

$$\boxed{K = 9 \times 10^9 \ Nm^2/C^2}$$

In SI unit, $\boxed{K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ Nm^2/C^2}$

$\epsilon_0$ = permittivity of free space

**PERMITTIVITY:-** Permittivity is the quantity that determines how far the medium permits the electrical interaction between two charged bodies.

$$\boxed{\epsilon_0 = 8.854 \times 10^{-12} \ C^2/Nm^2}$$

## CASE-1

In air/ vacuum/ free space :-

In SI :- $K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \, Nm^2/C^2$

$$F_{vac} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

## CASE-2

In any medium / dielectric medium

In SI :- $K = \frac{1}{4\pi\epsilon} = \frac{1}{4\pi\epsilon_0\epsilon_r} = \frac{1}{4\pi\epsilon_0 k}$

$$F_{med} = \frac{1}{4\pi\epsilon_0 k} \frac{q_1 q_2}{r^2}$$

$$\epsilon = \epsilon_0 \cdot \epsilon_r$$

$$F_{med} = \frac{F_{vac}}{k}$$

where, $\epsilon$ = permittivity of the medium/ electrical permittivity

$\epsilon_0$ = permittivity of free space

$\epsilon_r$ = dielectric constant (k)/ relative permittivity.

## ELECTRICAL PERMITTIVITY/ PERMITTIVITY OF THE MEDIUM :-

It is a constant defined for all mediums to know how far the medium permits the electrical interaction between two charged bodies.

- Symbol :- $\epsilon$
- Dimension :- $[M^{-1}L^{-3}T^4A^2]$

★ Also called as absolute permittivity.

## RELATIVE PERMITTIVITY ($\epsilon_r$) / DIELECTRIC CONSTANT (k)

The ratio of the permittivity of the medium to the permittivity of the free space is called relative permittivity ($\epsilon_r$) or dielectric constant (k).

$$\epsilon_r \text{ or } k = \frac{\epsilon}{\epsilon_0}$$

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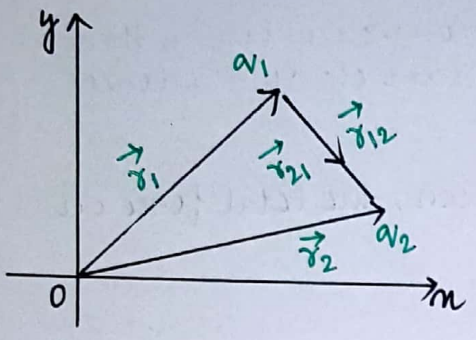
- Relative permittivity or dielectric constant has no unit and dimensionless.
- Symbol:- $\varepsilon_r$ or $k$
- For vacuum, $k = 1$
- For metal, $k = \infty$
- For water, $k = 80$

## COULOMB'S LAW IN VECTOR FORM:-




Force on $q_1$ due to $q_2$,

$$\vec{F}_{12} = \frac{K q_1 q_2}{r_{21}^2} \hat{r}_{21}$$

$$= \frac{K q_1 q_2}{r_{21}^2} \frac{\vec{r}_{21}}{r_{21}} = \frac{K q_1 q_2}{r_{21}^3} \vec{r}_{21}$$

$$= \frac{K q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)$$

Force on $q_2$ due to $q_1$,

$$\vec{F}_{21} = \frac{K q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

$$= \frac{K q_1 q_2}{r_{12}^2} \frac{\vec{r}_{12}}{r_{12}} = \frac{K q_1 q_2}{r_{12}^3} \vec{r}_{12}$$

$$= \frac{K q_1 q_2 (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3} = -\frac{K q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)$$

$$\boxed{\vec{F}_{21} = -\vec{F}_{12}}$$

This means that, the two charges exert equal & opposite force on each other. So, they obey Newton's third law of motion.

## CHARACTERSTICS OF COULOMB'S FORCE :-

1. Applicable or valid only for point charges which are at rest.
2. Obeys inverse square law ($F \propto \frac{1}{r^2}$)
3. It is a long range force.
4. Coulomb's force is inactive when the separation between two charges is less than one fermi ($10^{-15}$ m)
5. It is a central force i.e it act along the line joining the centres of the two bodies.

6. coulomb force depends on the medium within which charges are placed.

7. coulomb force is not affected by the presence of other charged bodies near it

8. It obeys newton's third law of motion

## FORCE BETWEEN MULTIPLE CHARGES :- THE SUPERPOSITION PRINCIPLE :-

When a number of charges are interacting among each other, then the force acting on one charge will be the vector sum of all the forces acting on it due to all other charges.

Then, according to the principle of superposition, the total force on charge $q_1$ is given by

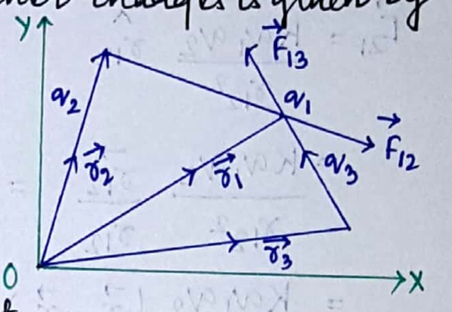
$$F_1 = F_{12} + F_{13} + \cdots + F_{1n} \quad \text{—— (1)}$$

$$F_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21}$$

Similarly, the force on charge $q_1$ due to other charges is given by

$$F_{13} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_3}{r_{31}^2} \hat{r}_{31}$$

$$F_{1n} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_n}{r_{n1}^2} \hat{r}_{n1}$$




Substituting, these values in eqn (1) we get,

$$F_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} + \frac{q_1 q_3}{r_{31}^2}\hat{r}_{31} + \cdots + \frac{q_1 q_n}{r_{n1}^2}\hat{r}_{n1}\right]$$

$$\boxed{F_{1i} = \frac{q_1}{4\pi\varepsilon_0} \sum_{i=2}^{n} \frac{q_i}{r_{i1}^2} \hat{r}_{i1}}$$

## ELECTRIC FIELD:-

The region surrounding to a charged body within which another charge experiences a force is called electric field.

## TEST CHARGE

→ The charge which produces the electric field is called source charge and the charge which experiences the effect of source charge is called test charge.

→ unit positive charge is taken as test charge.

→ its magnitude is very small in comparison to source charge because its own field shouldnot affect the field of source charge.

## ELECTRIC FIELD INTENSITY

It is defined as the force experienced per unit positive test charge placed at that point, without disturbing the source charge.

It is expressed as, $\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$ , where $\vec{E}$ = electric field intensity

$q_0$ = test charge

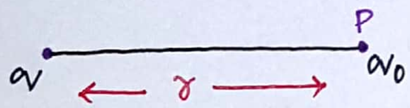
$\vec{F}$ = force experienced by the test charge $q_0$.

- It is a vector quantity
- SI unit :- N/C or V/m
- CGS unit :- D/stC or D/abC

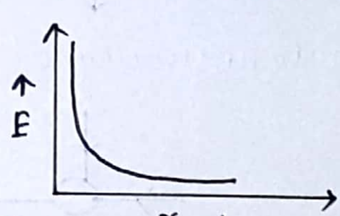
Dimension :- $\frac{M^1L^1T^{-2}}{AT} = [M^1L^1T^{-3}A^{-1}]$

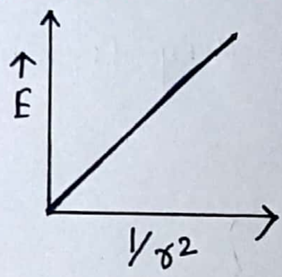
* Electric field due to positive charge is always away from it while due to negative charge is always towards it

## ELECTRIC FIELD INTENSITY DUE TO POINT CHARGE :-




P is any point at a distance r from the source charge q.




Force on $q_0$,

$$\vec{F} = \frac{Kqq_0}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{Kq}{r^2}\hat{r}$$

$$\boxed{|\vec{E}| = \frac{Kq}{r^2}}$$

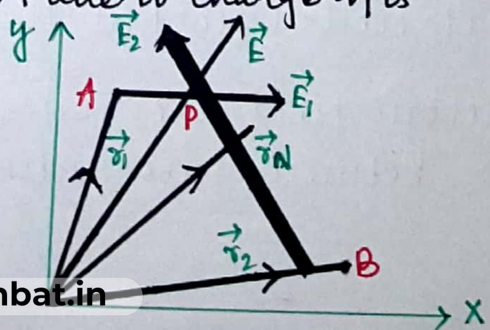
## ELECTRIC FIELD DUE TO MULTIPLE CHARGES :-

Consider $q_1, q_2, \ldots\ldots, q_N$ charges are placed at a dist $r_1, r_2, \ldots r_N$ from origin in vacuum. Hence, the electric field at point P due to charge $q_1$ is

$$\vec{E_1} = \frac{\vec{F_1}}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{1P}^2}\hat{r}_{1P}$$

Similarly,

$$\vec{E_2} = \frac{\vec{F_2}}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_{2P}^2}\hat{r}_{2P}$$

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$$\vec{E}_N = \frac{1}{4\pi\epsilon_0}\frac{q_N}{r_{NP}^2}\hat{r}_{NP}$$

According to the superposition principle,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots\cdots + \vec{E}_N$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{q_2}{r_{2P}^2}\hat{r}_{2P} + \cdots\cdots + \frac{q_N}{r_{NP}^2}\hat{r}_{NP}\right]$$

$$\Rightarrow \boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_{iP}^2}\hat{r}_{iP}}$$

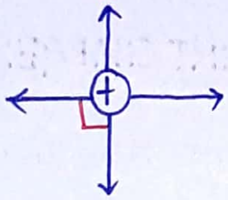
## ELECTRIC FIELD LINES / LINES OF FORCE:-

A curve along which the test charge would tend to move when force to do so in an electric field due to a source charge. These imaginary lines are called electric field lines.

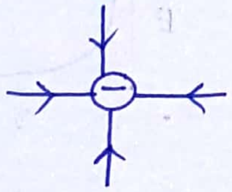
### PROPERTIES:-

① They start from positive charge and end at negative charge.

② They emerge normally from the surface of a positive charge.



③ They terminate normally on the surface of a negative charge.



④ The field lines have a tendency to expand laterally so as to exert a lateral pressure. This explains repulsion between two like charges.

⑤ Tangent to any point on electric field lines shows the direction of electric field at that point.

⑥ Electric field lines contract lengthwise to represent attraction between two unlike charges.

(7) Two field lines can never intersect each other because if they intersect, then two tangents drawn at that point will represent two directions of field at that point, which is not possible.

(8) They are continous smooth curve without any breaks

(9) They donot form closed loops.

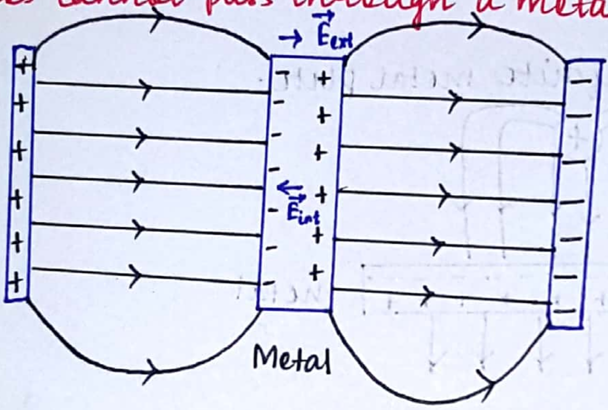
(10) The region where lines of force are crowded, its intensity is more.

(11) The number ΔN of lines per unit cross sectional area perpendicular to the field lines is directly proportional to the magnitude of the intensity of electric field in that region.

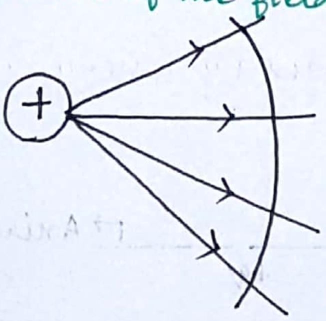
$$\frac{\Delta N}{\Delta A} \propto E$$

(12) They donot pass through a conductor

(13) Field lines cannot pass through a metallic slab.



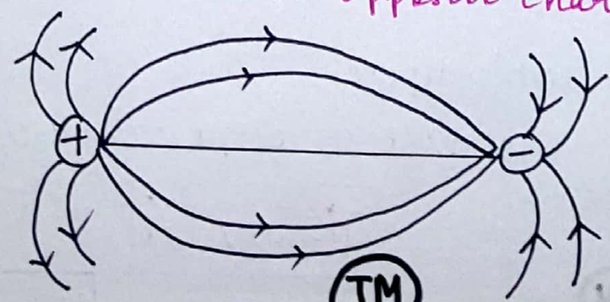

(14) The relative closeness of the field lines gives the strength of electric field



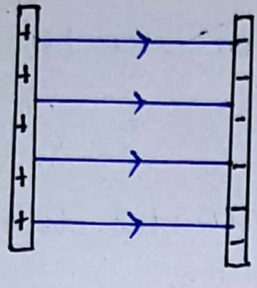
- Fields close to each other indicate strong field.
- Fields lines away to each other indicate weak field.

## REPRESENTATION OF ELECTRIC FIELD:-

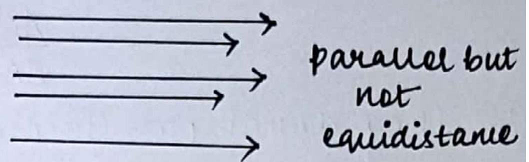
Electric field lines due to opposite charges are equal in magnitude.



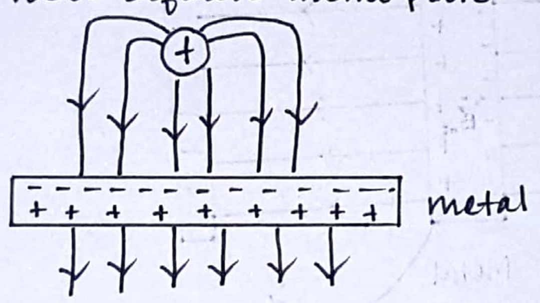
- In case of uniform field, the field lines are parallel (to have same direction) and are equidistance (to have same magnitude) to each other



- In case of non-uniform field, the field lines are not parallel and are not equidistance to each other.

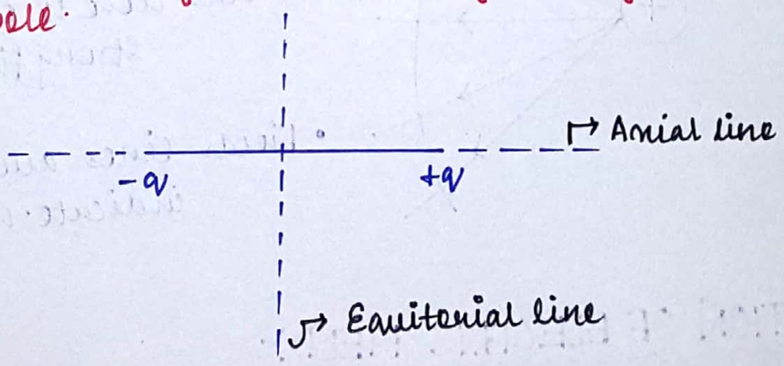



- Fixed point charge near infinite metal plate.




## ELECTRIC DIPOLE :-

Two equal and opposite charges separated by a very small distance constitute a dipole.




## ELECTRIC DIPOLE MOMENT :-

- It determines the strength of electric dipole
- It is define as the product of magnitude of either charge and separation of distance between them.

$$\vec{P} = q \times \vec{2l}$$

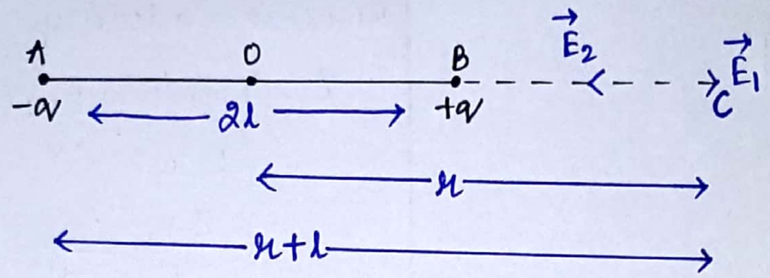
$$|P| = q(2l)$$

- vector quantity
- direction is always from negative charge to positive charge.
- Dimension – [ATL]
- SI unit – Cm

## IDEAL DIPOLE / POINT DIPOLE :-

Suppose, $q \to \infty$, $2l \to 0$ such that p is finite. Such a dipole of negligibly small size is called as ideal dipole or point dipole.

## ELECTRIC FIELD INTENSITY DUE TO DIPOLE AT THE AXIAL POSITION / END ON POSITION :-




C is any point on the axial line at a distance r from the centre of the dipole

Due to +q,

$$\vec{E}_1 = \frac{Kq}{BC^2}\hat{i}$$

$$= \frac{Kq}{(r-l)^2}\hat{i}$$

Due to −q,

$$\vec{E}_2 = \frac{Kq}{AC^2}(-\hat{i})$$

$$= \frac{Kq}{(r+l)^2}(-\hat{i})$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{Kq}{(r-l)^2}\hat{i} + \frac{Kq}{(r+l)^2}(-\hat{i})$$

$$= Kq\left[\frac{(r+l)^2 - (r-l)^2}{(r-l)^2\,(r+l)^2}\right]\hat{i}$$

$$= Kq\left[\frac{4rl}{(r^2-l^2)^2}\right]\hat{i}$$

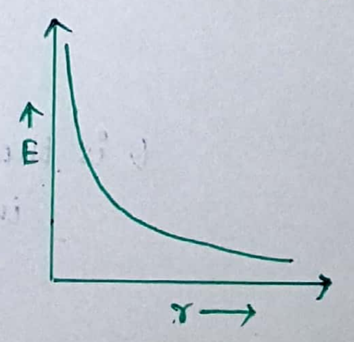
$$= Kq\,\frac{2r \cdot 2l}{(r^2-l^2)^2}\hat{i}$$

$$\boxed{\vec{E} = \frac{2Kpr}{(r^2-l^2)^2}\hat{i}}$$

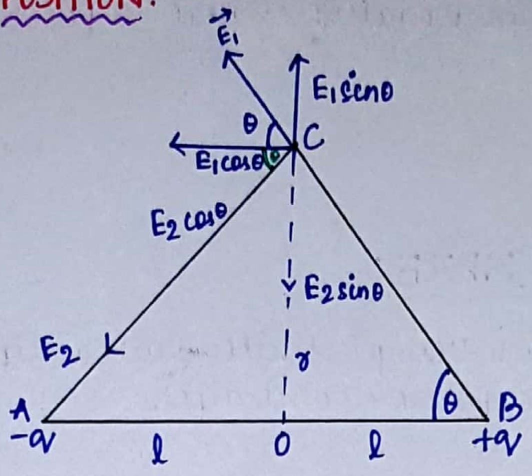
For ideal dipole, $l <<<< r$, so $l^2$ can be neglected

$$\vec{E} = \frac{2Kpr}{r^4}\hat{i}$$

$$\boxed{\vec{E} = \frac{2KP}{r^3}\hat{i}}$$



## ELECTRIC FIELD INTENSITY DUE TO DIPOLE AT AN EQUITORIAL POSITION OR BROAD SIDE ON POSITION:-




C is any point on the equitorial line at a distance r from the centre of the dipole.

Due to +q charge,

$$E_1 = \frac{Kq}{BC^2} = \frac{Kq}{r^2+l^2}$$

Due to -q charge,

$$E_2 = \frac{Kq}{AC^2} = \frac{Kq}{r^2+l^2}$$

$E_1 = E_2$

$E_1 \sin\theta$ and $E_2 \sin\theta$ cancel each other

Resultant Intensity,

$$\vec{E} = (E_1\cos\theta + E_2\cos\theta)(-\hat{i})$$

$$= 2E_1\cos\theta(-\hat{i})$$

$$= 2\frac{Kq}{r^2+l^2}\frac{l}{\sqrt{r^2+l^2}}(-\hat{i})$$

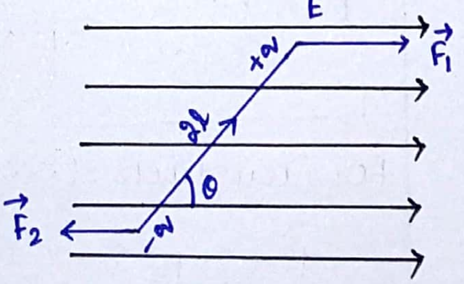
$$= \frac{2Kql}{(r^2+l^2)^{3/2}}(-\hat{i})$$

$$\boxed{\vec{E} = \frac{KP}{(r^2+l^2)^{3/2}}(-\hat{i})}$$

for ideal dipole, $l <<< r$, $l^2$ can be neglected

$$\boxed{\vec{E} = \frac{KP}{r^3}(-\hat{i})}$$

## DIPOLE IN UNIFORM ELECTRIC FIELD:-




$\theta$ is the angle between dipole moment and intensity.

Force on +q.

$\vec{F_1} = q\vec{E}$

Force on -q.

$\vec{F_2} = -q\vec{E}$

$\vec{F} = \vec{F_1} + \vec{F_2} = 0$

So no translatory motion

As two forces are not in same line of action, so they constitute a couple due to which dipole rotate.

$\tau = (2l) F \sin\theta$

$= 2lqE \sin\theta$

$= PE \sin\theta$

$$\boxed{\tau = PE\sin\theta}$$

In vector form,

$$\boxed{\vec{\tau} = \vec{P} \times \vec{E}}$$

$\vec{\tau} \perp \vec{P}$ and $\vec{\tau} \perp \vec{E}$

These are two pairs of perpendicular vector.

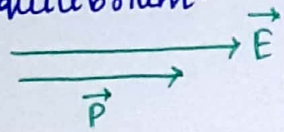
The direction of torque is $\perp^r$ to the plane inward according to figure.

**Case-1**

when $\theta = 0$

$\boxed{\tau = 0}$

It is a condition of stable equilibrium.

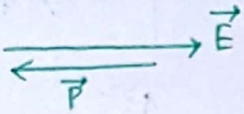



**Case-2**

when $\theta = 180^\circ$

$\boxed{\tau = 0}$

It is a condition of unstable equilibrium




**Case-3**

when $\theta = 90^\circ$

$\boxed{\tau = PE}$

Maximum torque

## NEUTRAL POINT :-

It is apoint in an electric field where when any charge is placed experience no force.

**CASE-1**

- for 2 like charges, neutral point lies between them. $+q_1$ N $+q_2$
- when similar charge is placed at neutral point, it is in unstable equilibrium along Y-axis and stable equilibrium along X-axis
- when dissimilar charge is placed at neutral point, it is in stable equilibrium along Y-axis and unstable equilibrium along X-axis.

**CASE-2**

- for two unlike charges, neutral point lies at the side of less magnitude charge.
- If $q_1 = q_2$, then neutral point is not possible.

# ELECTRIC FLUX :- ($\phi$)

## PHYSICAL SIGNIFICANCE :-

It determines the amount of electric field lines linked with the surface.

## DEFINATION :-

- It is defined as the dot product of electric field intensity with the arial vector of a surface.
- Electric flux at any point can be defined as the number of field line passing normally through that area placed inside an electric field.
- SI unit - $Nm^2/C$ or $Vm$
- Scalar quantity
- Dimension - $[ML^3T^{-3}A^{-1}]$

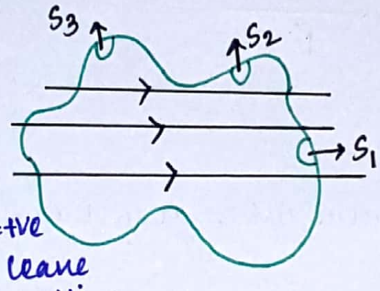
$$\phi = \vec{E} \cdot \vec{S}$$

$$\Rightarrow \boxed{\phi = ES\cos\theta}$$

$$\phi = \int \vec{E} \cdot \vec{ds}$$

$$\Rightarrow \boxed{\phi = \int E\,ds\cos\theta}$$

### Special case




**Case-1** (for surface $S_1$)

$\theta = <90°$, $\cos\theta = +ve$, flux $= +ve$

when the lines of force leave the surface, the flux is positive

**Case-2** (for $S_2$)

$\theta = 90°$, $\cos\theta = 0$

flux $= 0$

**Case-3** (for $S_3$)

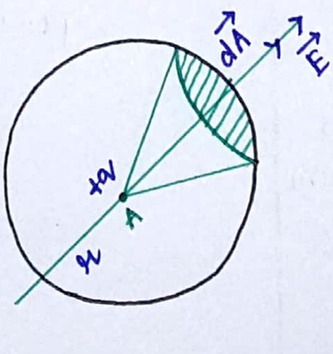
$\theta > 90°$, $\cos\theta = -ve$, flux $= -ve$

So, when lines of force enter to the surface, flux is negative

## GAUSS LAW :-

It states that the electric flux linked with a closed surface in vacuum is $\frac{1}{\epsilon_0}$ times the total charge enclosed within it.

### PROOF :-




Take a charge $+q$ at point A. Take a gaussian surface in the shape of a sphere of radius $r$ centred at $+q$

$$\phi = \oint E\,ds\cos\theta$$

$$= \oint E\,ds\cos 0$$

$$= E\oint ds$$

$$= \frac{Kq}{r^2} \times 4\pi r^2$$

$$= Kq4\pi$$

$$= \frac{1}{4\pi\epsilon_0} q4\pi$$

$$= \frac{q}{\epsilon_0}$$

$$\boxed{\phi = \frac{q_{en}}{\epsilon_0}}$$

JM

## DERIVATION OF COULOMB'S LAW FROM GAUSS LAW :-

According to gauss law,

$$\phi = \oint \vec{E} \cdot \vec{ds} = \frac{q_{en}}{\varepsilon_0}$$

$$\Rightarrow \oint E ds \cos 0 = \frac{q_{en}}{\varepsilon_0}$$

$$\Rightarrow E \oint ds = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{q}{4\pi r^2 \varepsilon_0} \quad \Rightarrow E = \frac{Kq}{r^2}$$

Suppose, a charge is placed on the periphery of the gaussian surface then force exerted on it will be,

$$F = E q_0$$

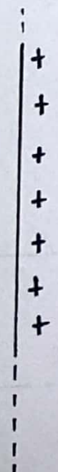
$$\boxed{F = \frac{K q q_0}{r^2}}$$

## IMPORTANT POINTS ON GAUSS LAW :-

- Gauss law is applicable for any closed surface, whatever its shape and size is.
- The surface in which gauss law is applied is called gaussian surface.
- Flux linked with closed surface is independent of area of the surface.
- If the medium is di-electric, then $\phi = \frac{q_{en}}{\varepsilon_0 k}$

## CONTINOUS CHARGE DISTRIBATION :-

(a) Linear charge distrubation :-



linear charge density,

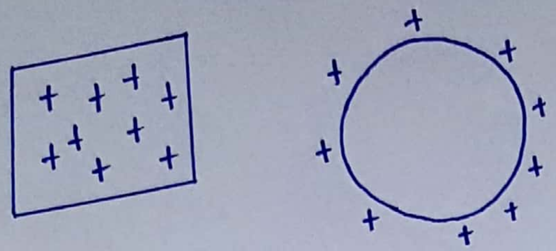
$$\lambda = \frac{q}{l}$$

$$\boxed{\lambda = \frac{dq}{dl}}$$

DIMENSION - $[A T L^{-1}]$

SI unit - C/m

(b) Surface charge distribution:-



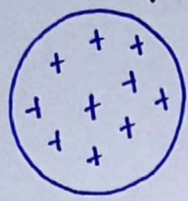
Surface charge density,

$$\sigma = \frac{q}{S} = \frac{dq}{dS}$$

DIMENSION - $[ATL^{-2}]$

SI unit - $C/m^2$

(C) Volume charge distribution:-



Volume charge density,

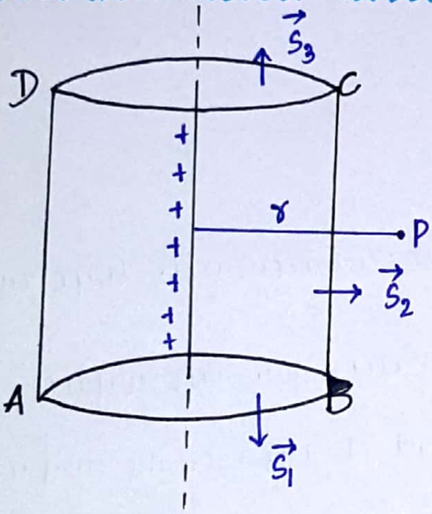
$$\rho = \frac{q}{V} = \frac{dq}{dV}$$

DIMENSION - $[ATL^{-3}]$

SI unit - $C/m^3$

## APPLICATION-1

(INFINITE LINE CHARGE / INFINITELY LONG CHARGED WIRE)




$\lambda$ = linear charge density = $\frac{q}{l}$

P is any point at a distance r from the line charge.

using gauss law,

$$\oint \vec{E}\cdot\vec{ds} = \frac{q_{en}}{\epsilon_0}$$

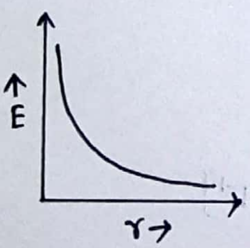
$$\Rightarrow \int_{S_1} \vec{E}\cdot\vec{ds} + \int_{S_2} \vec{E}\cdot\vec{ds} + \int_{S_3} \vec{E}\cdot\vec{ds} = \frac{q_{en}}{\epsilon_0}$$

$$\Rightarrow \int_{S_1} E\,ds\cos 90^\circ + \int_{S_2} E\,ds\cos 0 + \int_{S_3} E\,ds\cos 90^\circ = \frac{q_{en}}{\epsilon_0}$$

$$\Rightarrow E\cdot 2\pi r l = \frac{q_{en}}{\epsilon_0}$$

$$\Rightarrow E = \frac{q_{en}}{2\pi r l \epsilon_0}$$

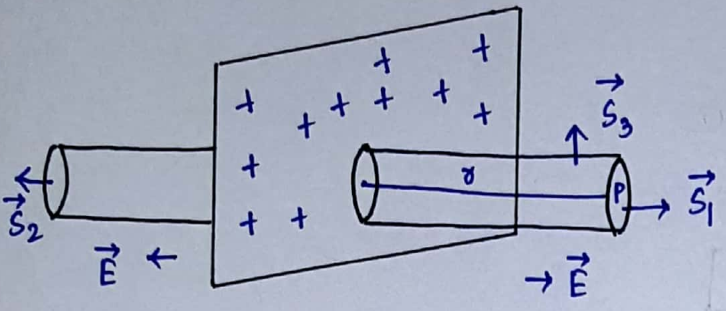
$$\Rightarrow \boxed{E = \frac{\lambda}{2\pi r \epsilon_0}}$$




In vector form, $\boxed{\vec{E} = \frac{\lambda}{2\pi r \epsilon_0}\hat{r}}$

## APPLICATION-2

### INFINITE THIN PLANE SHEET :-




σ = surface charge density

P is any point at a distance r from the plane sheet.

using gauss law,

$$\oint \vec{E}\cdot\vec{ds} = \frac{q_{en}}{\varepsilon_0}$$

$$\Rightarrow \int_{S_1} E\,ds\cos 0 + \int_{S_2} E\,ds\cos 0 + \int_{S_3} E\,ds\cos 90^\circ = \frac{q_{en}}{\varepsilon_0}$$

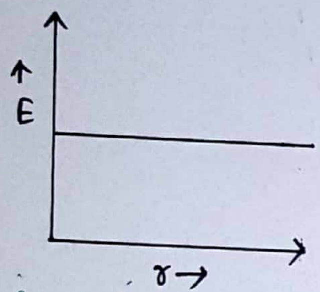
$$\Rightarrow ES + ES = \frac{q_{en}}{\varepsilon_0}$$

$$\Rightarrow 2ES = \frac{q_{en}}{\varepsilon_0}$$

$$\Rightarrow 2ES = \frac{\sigma S}{\varepsilon_0}$$

$$\Rightarrow 2E\cdot\varepsilon_0 = \sigma$$

$$\Rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

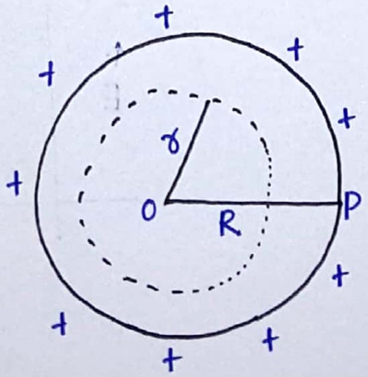



Electric field is independent of distance.

In vector form, $\boxed{\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}}$

## APPLICATION-3

### SPHERICAL SHELL (SOLID CONDUCTING SPHERE)




R = radius

$\sigma$ = surface charge density $= \frac{q}{4\pi R^2}$

Case-I

Inside

P is any point at a distance r from the centre ($r<R$)

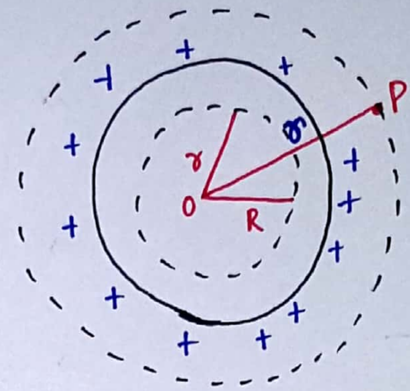
using gauss law, $\phi = \frac{q_{en}}{\epsilon_0}$

$\Rightarrow \oint \vec{E}\cdot\vec{ds} = 0$

$\Rightarrow$ $\boxed{E=0}$

Case-II ($r>R$)

Outside




Applying gauss law,

$\phi = \frac{q_{en}}{\epsilon_0}$

$\Rightarrow \oint \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$

$\Rightarrow \oint E\,ds\cos 0 = \frac{q}{\epsilon_0}$

$\Rightarrow E 4\pi r^2 = \frac{q}{\epsilon_0}$ $\Rightarrow$ $\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}}$

This expression is same as the expression used in intensity due to point charge. So charges resides on the surface of the spherical shell behave as if they are concentrated at the centre.

Case-III

On the surface ($r=R$)

using gauss law,

$\oint \vec{E}\cdot\vec{ds} = \frac{q_{en}}{\epsilon_0}$

$\Rightarrow \oint E\,ds\cos 0 = \frac{q}{\epsilon_0}$

$\Rightarrow E 4\pi R^2 = \frac{q}{\epsilon_0}$

$\Rightarrow E = \frac{q}{4\pi R^2\epsilon_0}$

$\Rightarrow$ $\boxed{E = \frac{\sigma}{\epsilon_0}}$